NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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CALCULATIONS OF THE EFFECT OF WING TWIST ON THE AIR FORCES ACTING ON A MONOPLANE WING*

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SUMMARY

A method is presented for calculating the aerodynamic forces on a monoplane wing, taking into account the elastic twisting of the wing due to these forces. The lift distribution along the span is calculated by the formulas of Amstutz as a function of the geometrical characteristics of the wing and of the twist at stations 60 and 90 percent of the semispan. The twist for a given lift distribution is calculated by means of influence lines. As a numerical example, the forces on a Swiss military D.27 airplane are calculated. Comparisons with the strip method and with the ordinary stress-analysis method are also given.

INTRODUCTION

Aerodynamic calculations on airplane wings are usually made by assuming that the wings are rigid bodies. In general, this method is allowable. In recently developed high-speed airplanes, however, with their increased wing loading, large deflecting forces act on the wing, especially in diving attitudes. The deformations, as a result of the air forces, influence these forces retroactively to such a degree that they cannot be neglected. The aerodynamic behavior of the deformed wing may be quite different from that of the original (rigid) one. The final deformations also become different from those resulting from the forces acting on the rigid wing.

It is of interest now to investigate these final deformations.

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*See also the preliminary publication on the same subject in the Schweizer Aero-Revue, Zurich-Oerlikon, October 15, 1931, p. 264; and Zeitschrift für Flugtechnik und Motorluftschiffahrt, January 28, 1932, p. 58.

I wish to thank Dr. J. Ackeret, Institute for Aerodynamics at the Eidg. Technische Hochschule, Zurich, for his interest in this research, shown at numerous times by his suggestions. As a private assistant of Professor L. Karner, at the same Hochschule, I became familiar with statics as applied to aircraft. Further, I wish to thank the Kreigstechnische Abteilung at Bern, from whom I was able to obtain the elastic data of the wing framework of D.27.

DEVELOPMENT OF THE PROBLEM

The calculation of the final deformations, considering the mutual relation between wing forces and deformations, involves two fundamental problems:

1. Calculation of the lift distribution along the span of the deformed wing;

2. Calculation of the wing deformations for given lift forces.

There are different types of wing deformation. Among them, the wing twisting is of predominating importance since the air forces are especially sensitive to changes in angle of attack. The problem may therefore be restricted to the one of elastic wing twisting.

For a given twist of the wing, the air forces are calculated for a given angle of attack and a given dynamic pressure. From them follow certain angles of wing twist. The final twist depends upon the condition that the assumed twist will lead to the same resulting twist.

Aerodynamic Fundamentals

The symbols used in the remaining sections of the paper are listed here for reference.

x, distance of any point or section along the wing span from the center wing section (plane of symmetry).

b, wing span.

ξ, $\frac{x}{b/2}$

t, wing chord at the distance x or ξ.

V, speed of flight.

w, vertical downwash velocity induced by the trailing vortices.

α, angle of attack.

δ, angle of twist.

ρ, mass density of air.

$q = \frac{\rho}{2} V^2$, dynamic pressure.

S, wing area.

L, lift of the wing.

D, drag of the wing.

D_p, profile drag of the wing.

D_i, induced drag of the wing.

M, moment of the wing air forces with respect to the profile leading edge.

$C_L = \frac{L}{q\,S}$, absolute lift coefficient.

$C_D = \frac{D}{q\,S}$, absolute drag coefficient.

$C_{D_p} = \frac{D_p}{q\,S}$, absolute profile-drag coefficient.

$C_{D_i} = \frac{D_i}{q\,S}$, absolute induced-drag coefficient.

$C_M = \frac{M}{q\,S\,t}$, absolute pitching-moment coefficient.

Γ, circulation as defined by

$$\Gamma = \frac{\text{lift per meter length of span}}{\rho V} = \frac{c_L}{2} V t$$

μ, ν, absolute dimensionless coefficients.

All values regarding the center wing section ($x = \xi = 0$) are denoted by the subscript zero.

In addition, there is the symbol α_∞, geometrical angle of attack of the wing measured with respect to the flight direction. The symbol ∞ is used because the wing with infinite span has no induced downwash velocities. Then the geometrical and the effective angles of attack are the same. Thus c_{L_∞} is the lift coefficient corresponding to the geometrical angle of attack α_∞. The connection between α_∞ and c_{L_∞} is given by the polar of the wing section (wing profile) for infinite wing span.

Now, as to the first fundamental problem, i.e., the calculation of the air forces (lift distribution along the wing span), the formulas developed by E. Amstutz (reference 1) are used. They start from Prandtl's formula for the lift distribution

$$\frac{\Gamma}{\Gamma_0} = \sqrt{1 - \xi^2}\left\{1 + \mu\,\xi^2 + \nu\,\xi^4 + \ldots\right\}$$

Hereby additional lift distributions are superimposed upon the original elliptic distribution. The coefficients μ, ν.... give the magnitude of their ratio.

Following Amstutz' formulas, which consider only the first two coefficients μ and ν, the approximate lift distribution of a given wing may be calculated, at a given angle of attack and a given dynamic pressure, as a function of the geometrical data and the angles of twist of two sections along the wing span. They advantageously are assumed at about 50 percent and 90 percent of the wing semispan. As shown by J. Knaber of Gottingen (reference 2), the method of Amstutz (reference 1) gives very good results in comparison with the more accurate method developed by I. Lotz (reference 3). (If the method of Lotz were used, the curve showing the wing twist along the wing span would have to be assumed a priori. The method of Amstutz, however, is more general in this respect. By means of it both the curve of the wing twist along the wing span

and the values of the angles of twist may be calculated without the foregoing assumption.)

Static Fundamentals

The second fundamental problem, the calculation of the wing twist for a given lift distribution, is treated by means of lines of influence for the wing twist. They are the results of an extended research (interaction between the spars due to the ribs) on the wing framework of the Swiss military fighter D.27, shown in figure 1, and fully described in reference 4. This fighter has an all-metal aluminum framework wing with two parallel and identical spars.

The ordinates of the lines of influence give the angles of twist along the wing span for any given single vertical load of 100 kilograms acting at two points of one of the spars, these two points being symmetrically placed with regard to the center wing section. For given distributed loads acting on both spars at the same time, the ordinates of the curve showing the difference between the load on the front spar and that on the rear must be multiplied by the ordinates of the lines of influence. Then the angles of wing twist are given as the integrals of the curves obtained by the above multiplication. For equal loads on both spars acting in the same direction pure wing bending without any twisting is obtained.

The lines of influence are shown in figure 2. As the wing is of the semicantilever type (a so-called "parasol" wing), the lines of influence were formerly referred to the points where the struts are attached. In this calculation, however, these influence lines must be referred to the center wing section because of the requirements of the aerodynamic calculations. That change in reference was accomplished by subtracting the ordinates of the influence line belonging to the center wing section from the corresponding ordinates of the influence lines belonging to the other sections. In that way the angle of twist at the center wing section always becomes equal to zero and the wing seems to be a real cantilever one. The line of influence of the section where the struts are attached now is the same as the earlier line of influence of the center wing section, except that the sign is changed.

THE CALCULATIONS

The general ideas having now been given, the calculations using the D.27 as an example, follow.

The wing plan form is changed slightly into an elliptical one with nearly the same area (fig. 3)

$$S = 17.8 \text{ m}^2$$

and exactly the same span

$$b = 10.3 \text{ m}$$

as that of the D.27, the mean chord being

$$t_o = 2.20 \text{ m}$$

Further, a plane wing is assumed without any original twist, having the same profile along the entire span, preserving this profile for any wing deformations. Thus a pure elliptical lift distribution is obtained on the plane wing. The wing plan form is placed in such a way that the distance s of the center line between the two spars from the leading edge of the wing has the constant value of 37.15 percent of the wing chord. This is done in order to simplify the calculations. As the static wing-cell structure is symmetrical, the center line is the so-called "elastic wing axis." Any load acting along that axis produces pure wing bending as both the spars bend identically.

The profile Göttingen 398 (see reference 5) is assumed. Figure 4 shows this profile together with its polar for infinite span. We obtain the equation

$$C_L = 5.2042 \, (\alpha_B + 0.1149)$$

where α_B is the angle of attack with respect to the profile chord, measured in radians.

The slope $\dfrac{dC_L}{d\alpha} = 5.2042 = k.$

Further $C_m = 1.2615 \, (\alpha_B + 0.18569)$

$$C_m = 0.2424 C_L + 0.06982$$

Values of $C_{D_p} = f(C_{L_\infty})$ for large angles of attack may be found in reference 5.

The Calculation of the Angles of Twist

δ_A and δ_S

By means of the lines of influence, their ordinates being τ, the general expression for the angles of wing twist

$$\delta = \int_0^{t/2} \tau \, \Delta p \, dx$$

is obtained, Δp being the difference between the load per unit length of the front spar p_V and the load for unit length of the rear spar p_H, i.e., $\Delta p = p_V - p_H$. The vertical wing load is approximately equal to the wing lift per unit length of span (assuming the cosine of the angle of attack equal to unity),

$$p \sim \frac{dL}{dx} = \Gamma \rho V .$$

The resultant of p having its point of application at an approximate distance

$$e = \frac{C_m}{C_L} t$$

from the leading edge of the wing.

The wing lift is now distributed between both the spars. Therefore (see fig. 5)

$$p_V + p_H = \frac{dL}{dx} = \Gamma \rho V.$$

Each element of the wing contributes to the moment M with

$$dM = p_V \, dx \, v + p_H \, dx \, h$$

or

$$\frac{dM}{dx} = p_V \, v + p_H \, h.$$

On the other side

$$dM = C_m\, q\, t\, dS = C_m\, q\, t^2\, dx$$

and

$$\frac{dM}{dx} = C_m\, q\, t^2.$$

Hence $\quad C_m\, q\, t^2 = p_V\, v + p_H\, h$

Putting $\quad v = h - d$

$$p_V + p_H = \Gamma \rho V$$

results in

$$p_V = \frac{h\,\Gamma\rho V - C_m\, q\, t^2}{d}$$

In the same way

$$p_H = \frac{C_m\, q\, t^2 - v\Gamma\rho V}{d}$$

With

$$\left.\begin{aligned} q &= \frac{\rho}{2} V^2 \\ v + h &= 2S \end{aligned}\right\}$$

$$p_V - p_H = \Delta p = \frac{\rho V}{d} \left\{ \Gamma 2S - C_m\, V\, t^2 \right\}$$

Now

$$\left.\begin{aligned} \Gamma &= \frac{C_L}{2} V\, t \\ C_m &= 0.2424\, C_L + 0.08922 \end{aligned}\right\}$$

$$2S = 0.743\ t$$

hence $\quad \Delta p = q\, \frac{t^2}{d} \left\{ 0.2582\, C_L - 0.1796 \right\}$

As developed by Amstutz.

$$C_L = \frac{2\Gamma_o}{V\, t} \left\{ \sqrt{1 - \xi^2}\ \left\{ 1 + \mu\, \xi^2 + \nu\, \xi^4 \right\} \right.$$

and

$$\Gamma_o = \frac{C_{L_{\infty}}\, V\, t_o}{2 \left\{ 1 + \frac{k\, t_o}{4b} \left[1 - \frac{\mu}{2} - \frac{\nu}{8} \right] \right\}}$$

By putting $t^2 = t_0^2 \left\{1 - \xi^2\right\}$

$$c_L = c_{L_{m_0}} \frac{(1 + \mu\,\xi^2 + \nu\,\xi^4)}{\left\{1 + \frac{k\,t_0}{4b}\,[1 - \frac{\mu}{2} - \frac{\nu}{8}]\right\}}$$

Hence

$$\Delta p = q\,\frac{t_0^2}{2}\,(1 - \xi^2)\left\{\frac{0.2582\,c_{L_{m_0}}\,(1 + \mu\,\xi^2 + \nu\,\xi^4)}{1 + \frac{k\,t_0}{4b}\,[1 - \frac{\mu}{2} - \frac{\nu}{8}]} - 0.1798\right\}$$

From this is obtained the even power series

$$\Delta p = K^* + L^*\,\xi^2 + M^*\,\xi^4 - N^*\,\xi^6$$

the coefficients K^*, L^* .... containing all the terms independent of ξ. Hence,

$$\delta = \int_0^{b/2} \tau\left\{K^* + L^*\,\xi^2 + M^*\,\xi^4 - N^*\,\xi^6\right\} dx$$

Replacing dx by $\frac{b}{2}\,d\xi$ and considering that the ordinates τ are based on 100 kg as a unit, and with

$$K^*\,\frac{b}{200} = K \quad L^*\,\frac{b}{200} = L$$

$$M^*\,\frac{b}{200} = M \quad N^*\,\frac{b}{200} = N$$

there is obtained

$$\delta_3 = K\int_0^1 \tau_3\,d\xi + L\int_0^1 \tau_3\,\xi^2\,d\xi + M\int_0^1 \tau_3\,\xi^4\,d\xi - N\int_0^1 \tau_3\,\xi^6\,d\xi$$

$$\delta_5 = K\int_0^1 \tau_5\,d\xi + L\int_0^1 \tau_5\,\xi^2\,d\xi + M\int_0^1 \tau_5\,\xi^4\,d\xi - N\int_0^1 \tau_5\,\xi^6\,d\xi$$

The curves $[\tau\,\xi^{2n}]$ are shown in figure 2. By graphical integration

$$\delta_3 = 0.12447\,K + 0.06447\,L + 0.03303\,M - 0.02555\,N$$

$$\delta_5 = 0.23850\,K + 0.16520\,L + 0.10370\,M - 0.07184\,N$$

Replacing K, L, M, and N by their expressions

$$\delta_3 = \frac{q}{100}\left\{\frac{C_{L_{\infty_0}}\,[0.5149 + 0.2183\,\mu + 0.1157\,\nu]}{1 + \frac{k\,t_0}{4b}\left[1 - \frac{\mu}{2} - \frac{\nu}{8}\right]} - 0.3582\right\}$$

$$\delta_5 = \frac{q}{100}\left\{\frac{C_{L_{\infty_0}}\,[1.0495 + 0.5363\,\mu + 0.2734\,\nu]}{1 + \frac{k\,t_0}{4b}\left[1 - \frac{\mu}{2} - \frac{\nu}{8}\right]} - 0.7302\right\}$$

The coefficients μ and ν for generally given values δ_3 and δ_5 must be calculated. Anstutz developed the linear equations

$$A_1 - \mu\,B_1 - \nu\,C_1 = 0$$

$$A_2 - \mu\,B_2 - \nu\,C_2 = 0$$

where

$$A_1 = \frac{C_{L_{\infty_1}}\,t_1}{C_{L_{\infty_0}}\,t_0} + \frac{kt_1}{4b}\left(\frac{C_{L_{\infty_1}}}{C_{L_{\infty_0}}} - 1\right) - \sqrt{1 - \xi_1^{\,2}}$$

$$B_1 = \frac{kt_1}{8b}\left(\frac{C_{L_{\infty_1}}}{C_{L_{\infty_0}}} - 1\right) + \frac{kt_1}{4b}\,3\,\xi_1^{\,2} + \xi_1^{\,2}\sqrt{1 - \xi_1^{\,2}}$$

$$C_1 = \frac{kt_1}{32}\left(\frac{C_{L_{\infty_1}}}{C_{L_{\infty_0}}} - 1\right) + \frac{kt_1}{4b}\left(5\,\xi_1^{\,4} - \frac{3}{2}\,\xi_1^{\,2}\right) + \xi_1^{\,4}\sqrt{1 - \xi_1^{\,2}}$$

A_2, B_2, and C_2 are deduced by replacing $C_{L_{\infty_1}}$, t_1, and ξ_1 by $C_{L_{\infty_2}}$, t_2, and ξ_2.

Since $\quad \xi_1 \longrightarrow \xi_3 \qquad \xi_2 \longrightarrow \xi_5$

the equations become

$$A_3 - \mu\,B_3 - \nu\,C_3 = 0$$

$$A_5 - \mu\,B_5 - \nu\,C_5 = 0$$

The values $C_{L_{m_3}}$ and $C_{L_{m_5}}$ contained in the terms A, B, and C are given by the values $C_{L_{m_0}}$ of the center wing section and the angles of twist δ_3 and δ_5, i.e.

$$C_{L_{m_3}} = C_{L_{m_0}} + k\,\delta_3$$

$$C_{L_{m_5}} = C_{L_{m_0}} + k\,\delta_5$$

The expressions A, B, and C are calculated first, then the coefficients μ and ν as functions of δ_3 and δ_5, and of $C_{L_{m_0}}$. Then inserting the values for μ and ν into the equations for δ_3 and δ_5, there is obtained, after extended transformations

$$\delta_3 = \frac{q}{100}\left\{0.0229\,\delta_3 + 0.0083\,\delta_5 + 0.4029\,C_{L_{m_0}} - 0.3582\right\}$$

$$\delta_5 = \frac{q}{100}\left\{0.0557\,\delta_3 + 0.0190\,\delta_5 - 0.8213\,C_{L_{m_0}} - 0.7302\right\}$$

The solutions are

$$\delta_3 = \frac{C_{L_{m_0}}\left\{\dfrac{4871.621}{q} - 0.1067\right\} - \dfrac{4331.227}{q} + 0.0949}{\dfrac{1209043}{q^2} - \dfrac{507.169}{q} - 0.003002}$$

$$\delta_5 = \frac{C_{L_{m_0}}\left\{\dfrac{1472.349}{q} + 0.0555\right\} - \dfrac{1309.924}{q} - 0.0583}{\dfrac{179401.1}{q^2} - \dfrac{75.255}{q} - 0.000445}$$

Neglecting the small values in numerator and denominator, the simple formulas are derived

$$\delta_3 = \frac{9.2339\,C_{L_{m_0}} - 8.2134}{\dfrac{2351.09}{q} - 1}$$

$$\delta_s = \frac{19.9363\ C_{L_{w_0}} - 17.750}{\dfrac{2351.09}{q} - 1}$$

These angles are given in degrees, and since it is assumed that $\Delta p > 0$ when $p_V > p_H$, they have the same sign as that of the angle of attack.

Both the angles depend linearly on $C_{L_{w_0}}$, i.e., on the geometrical angle of attack α_{w_0} at the center wing section. (See figs. 6 and 7.) Furthermore, at any dynamic pressure q, they become equal to zero when $C_{L_{w_0}} = 0.890$, i.e., $\alpha_{w_0} = 3.22°$. At this angle of attack the wing preserves its original untwisted shape. At $q =$ 2351.09 kg/m², i.e., at a speed of about 700 km/h at sea level, the angles of wing twist evidently become infinite for all angles of attack of the center wing section. This value of dynamic pressure represents the limit of the static torsional stability of the wing.[1] (See also references 7 and 8.)

The ratio of magnitude between δ_a and δ_s is constant and has the value

$$\frac{\delta_a}{\delta_s} = 2.161$$

Hence it is deduced that the curve showing the distribution of the angles of wing twist along the wing span will always be the same.

As an example

$$\delta_s = -\ 2.3° \qquad \delta_a = -\ 5.0°$$

for $C_{L_{w_0}} = 0.3$; $q = 625$ kg/m², i.e., $V = 360$ km/h at

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[1] The author wishes to acknowledge the kindness of Dr. C. Minelli for the interest he showed in the research by furnishing several Italian papers.

sea level. As will be noted (fig. 8), the angle of twist at the wing tips becomes 20 percent larger than δ_6. Therefore, the wing tips may easily fall into negative stalled flight. Since the flow detaches itself from the lower side of the wing tips at this moment, there will be a considerable change in air forces, which may give an impulse to wing oscillations. This explanation is suggested by some researches recently made by J. Ackeret and H. L. Studer (reference 8).

The Polar of the Elastic Wing

The knowledge of the values δ_5 and δ_6 enables the calculation of the values μ and ν. The lift distributions may therefore be calculated along the wing span as a function of $C_{L_{w_0}}$, i.e., of C_{w_0}, and of the dynamic pressure q. The angles of wing twist along the wing span is also obtained as was previously shown, by the integral

$$\delta_n = \int_0^{b/2} \Delta p \, \tau_n \, dx$$

introducing now the different lines of influence with their ordinates τ_n. These integrals are evaluated to find the values (see fig. 8)

$$\frac{\delta_1}{\delta_6} = 0.0586 \qquad \frac{\delta_2}{\delta_6} = 0.2028$$

$$\frac{\delta_3}{\delta_6} = 0.4628 \qquad \frac{\delta_4}{\delta_6} = 0.7390$$

The lift distribution along the wing span being known for any condition of flight, all the local aerodynamic values (C_L, C_D, C_m) along the wing span may be calculated. Hence the lift of the entire wing with its absolute coefficient $\overline{C_L}$, and the profile drag represented by the coefficient $\overline{C_{D_p}}$ is obtained, both as average values of all the local values C_L and C_{D_p}. Finally the induced drag of the wing is represented by the coefficient C_{D_i}. (See formulas developed by Amstutz.) All these values en-

able the plotting of the polars of the elastic wing. (See figs. 9 and 10.) The induced polars in figure 10 are shown by dashes. The polars that include the profile drag are solid.

All the polars go through the point $\overline{C_L} = 0.698$, corresponding to the value $C_{L_{x_0}} = 0.589$, where the wing preserves its original untwisted shape. (See reference 10.) Evidently the wing twist is appreciable only at small angles of attack, i.e., at small values of $\overline{C_L}$, corresponding either to level flight with full speed or especially to diving. It may also be mentioned that for the elastic wing, the value $\overline{C_L} = 0$ does not correspond to the condition in which the local values of the lift coefficient are equal to zero, as was the case with the original (rigid) wing. But now the center wing portion produces a positive lift that is compensated for by negative lift at the wing tips. Therefore, the semicantilever wing may be stressed by additional bending to such a degree that it may even become dangerous. On the other hand, the bending stress may become smaller in the case for which all local values of C_L are either positive or negative. This is especially true for the full cantilever wing, as the wing load is concentrated at the center wing section.

The polars also include the $\overline{C_m}$ curves of the elastic wing. These coefficients of pitching moment result from the integral

$$\overline{C_m} = \frac{M}{q\,S\,t_0} = \frac{1}{q\,S\,t_0} \int_{-b/2}^{b/2} C_m\, q\, t\, dS$$

from which

$$\overline{C_m} = \frac{4}{\pi} \int_0^1 C_m\,(1 - \xi^2)\, d\xi$$

by a consideration of

$$S = \frac{\pi}{4}\, b\, t_0 \qquad dS = t\, dx \qquad t = t_0\sqrt{1 - \xi^2}$$

$$dx = \frac{b}{2}\, d\xi \qquad \int_{-1}^{1} = 2\int_0^1$$

The numerical calculations show that for $\overline{C_L}$ = constant, there is almost no change in $\overline{C_m}$ for different values of the dynamic pressure q, which indicates that the wing moment M for $\overline{C_L}$ = constant is very slightly influenced by the wing twist. (See also references 11 and 12 and bibliography of reference 11.) The relation

$$\overline{C_m} = 0.2058\ \overline{C_L} + 0.0762$$

which was used at first only for the rigid wing may also be used for the elastic wing.

The Angles of Wing Twist When No Changes in Downwash Velocities are Considered (Strip Method)

It may be interesting to give the results of the approximate calculation of wing twist by the strip method.

In this case the lift distribution is given by the geometrical angles of attack as shown in the polar of the rigid wing with the given aspect ratio

$$A_1 R_0 = 5.96$$

These angles of wing twist differ from the former accurate values by about -5 percent to 15 percent. The outline of their curve along the wing span is given in the following:

$$\frac{\delta_1}{\delta_5} = 0.0721 \qquad \frac{\delta_2}{\delta_5} = 0.199$$

$$\frac{\delta_3}{\delta_5} = 0.449 \qquad \frac{\delta_4}{\delta_5} = 0.703$$

The critical dynamic pressure becomes 2,025.70 kg/m², which occurs at about 650 km/h at sea level.

COMPARISON WITH THE USUAL STRESS CALCULATION

In the usual stress calculation the angles of wing twist are also given by the integral

$$\delta_n^* = \int_0^{b/2} \Delta p^* \tau_n \, dr$$

But Δp^* now has to be calculated by introducing the forces acting on the rigid wing, as no influence of the wing twist upon the air forces is considered.

The results are

$$\frac{\delta_1^*}{\delta_5^*} = 0.0675 \qquad \frac{\delta_2^*}{\delta_5^*} = 0.2096$$

$$\frac{\delta_3^*}{\delta_5^*} = 0.430 \qquad \frac{\delta_4^*}{\delta_5^*} = 0.764$$

The outline of this curve is practically the same as the one which was obtained by the accurate method. Hence the outline of the curve showing the wing twist along the span in the example is the same, whether or not the interaction between wing twist and air forces is considered.

The relation between these values and the accurate ones is given by the fact that these approximate values represent the tangents on the curves of the accurate values at $q = 0$. If q_m be the critical value of the dynamic pressure of the static torsional stability, there is obtained the relation

$$\delta = \delta^* \left\{ \frac{1}{1 - \frac{q}{q_m}} \right\}$$

The value in the parenthesis gives the increase of the angles of wing twist due to the mutual interference between deformations and air forces.

The results presented are those of the present research. They require further checking to show how valid they may be for cases more general than that of the example.

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Figure 1. - Wing framework of the Swiss military fighter airplane D27

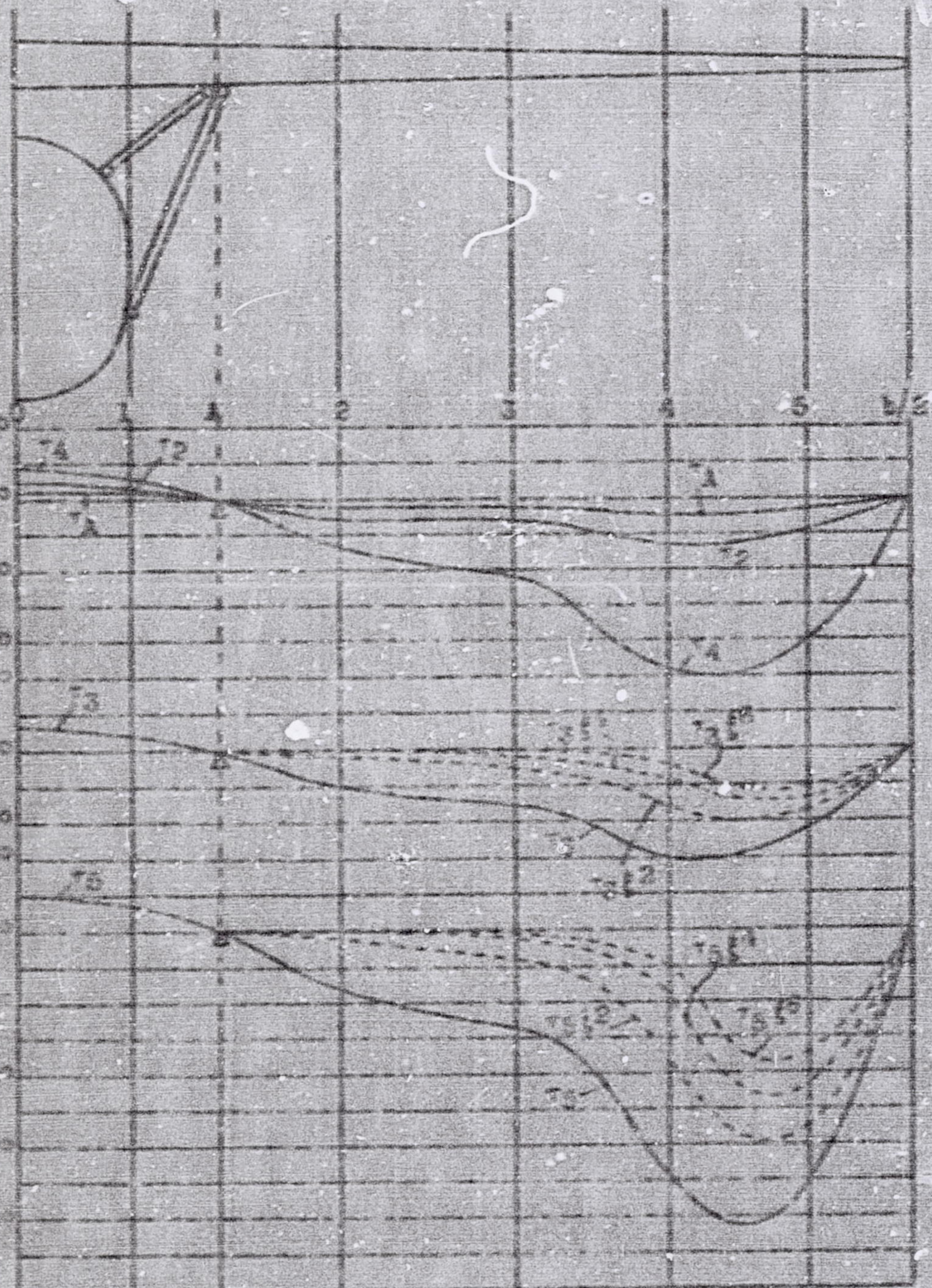


Figure 2.- Lines of influence of the wing twist from D.27 airplane

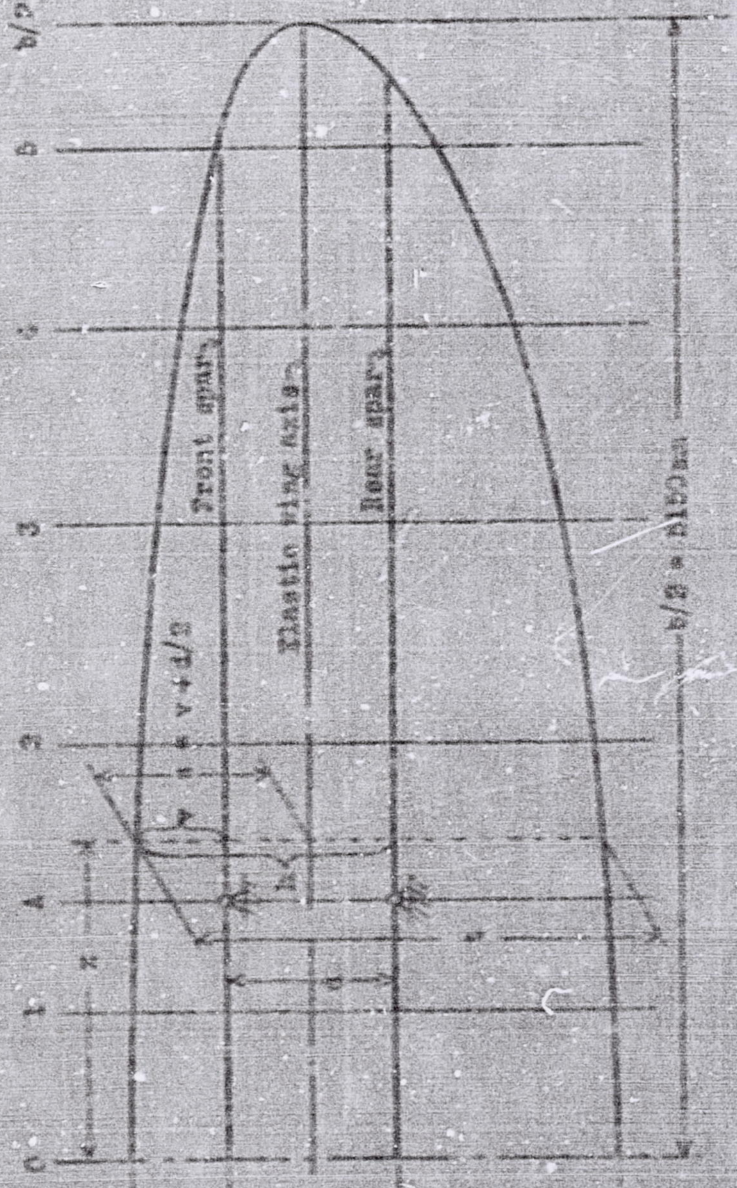


Figure 3.-Elliptical wing with approximately the same area as the standard wing of the D.27 airplane.

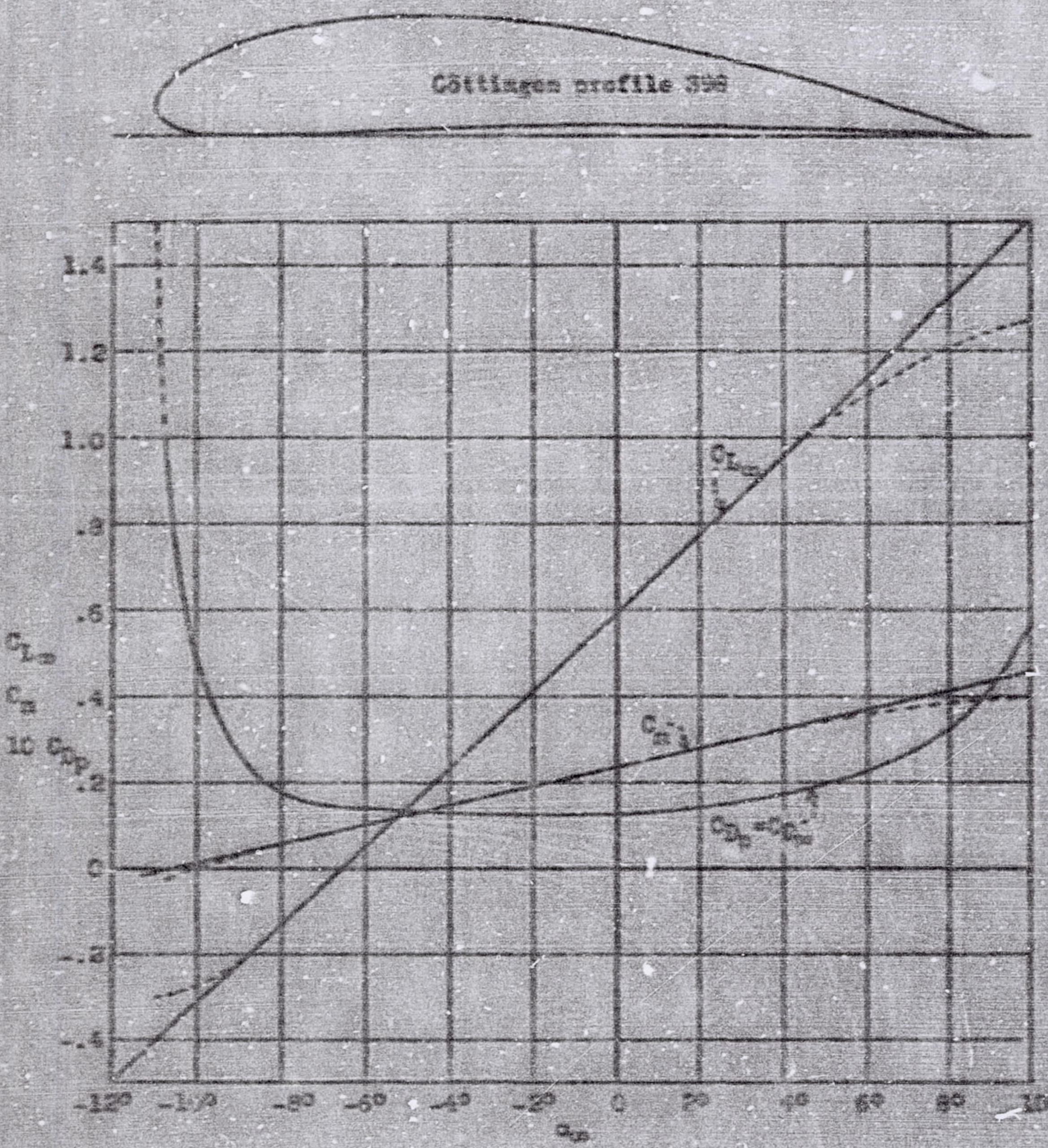


Figure 4.-The profile and polar of the Göttingen 398 airfoil.

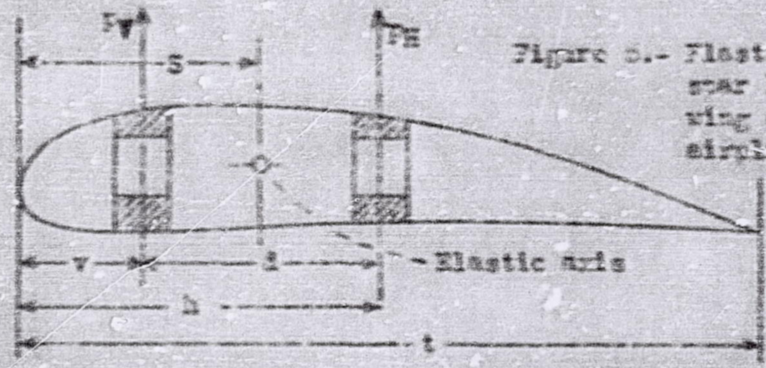


Figure 5.- Elastic axis and spar location of wing of D.27 airplane.

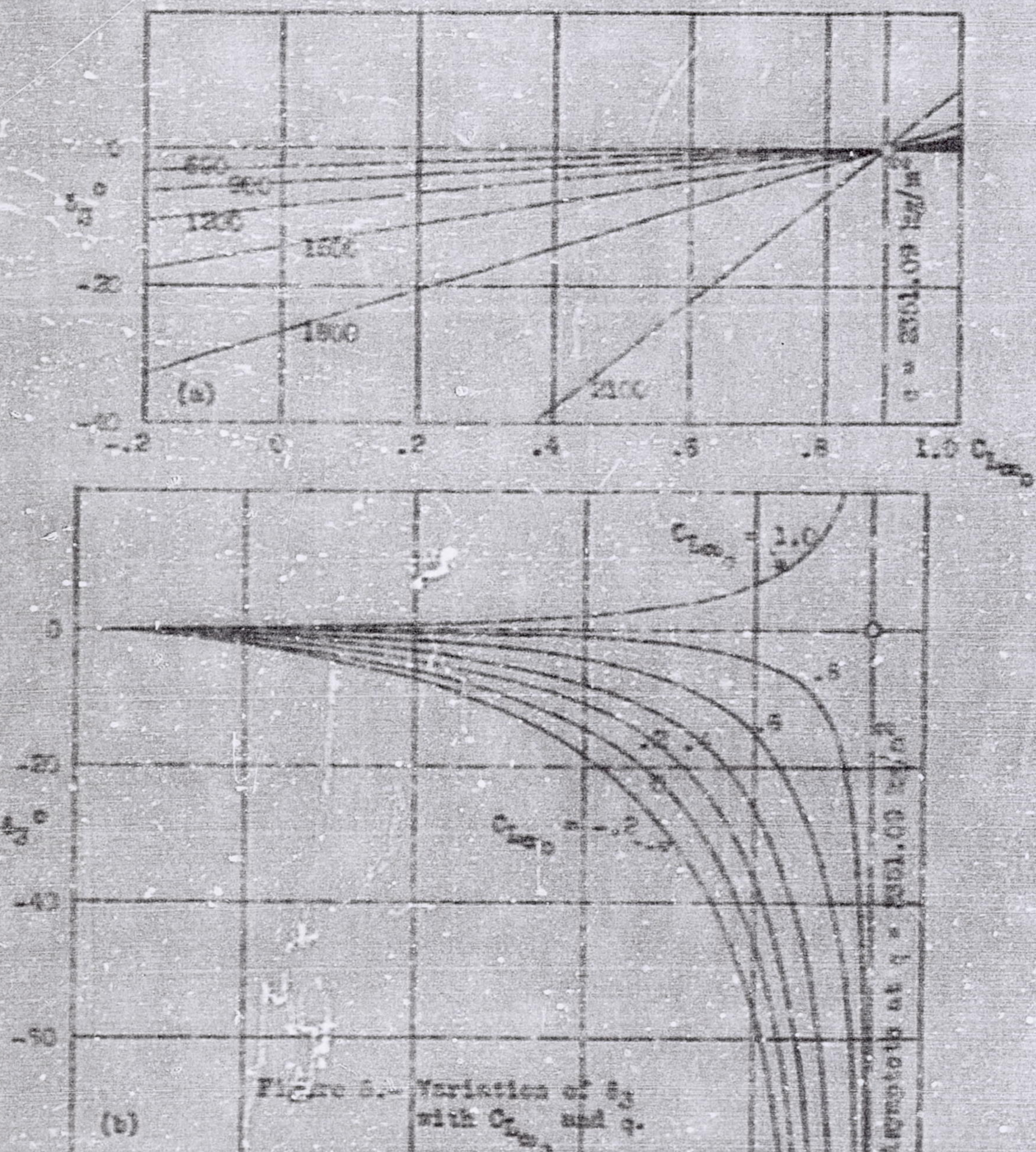


Figure 6.- Variation of δ_3 with $C_{L_{m_0}}$ and q.

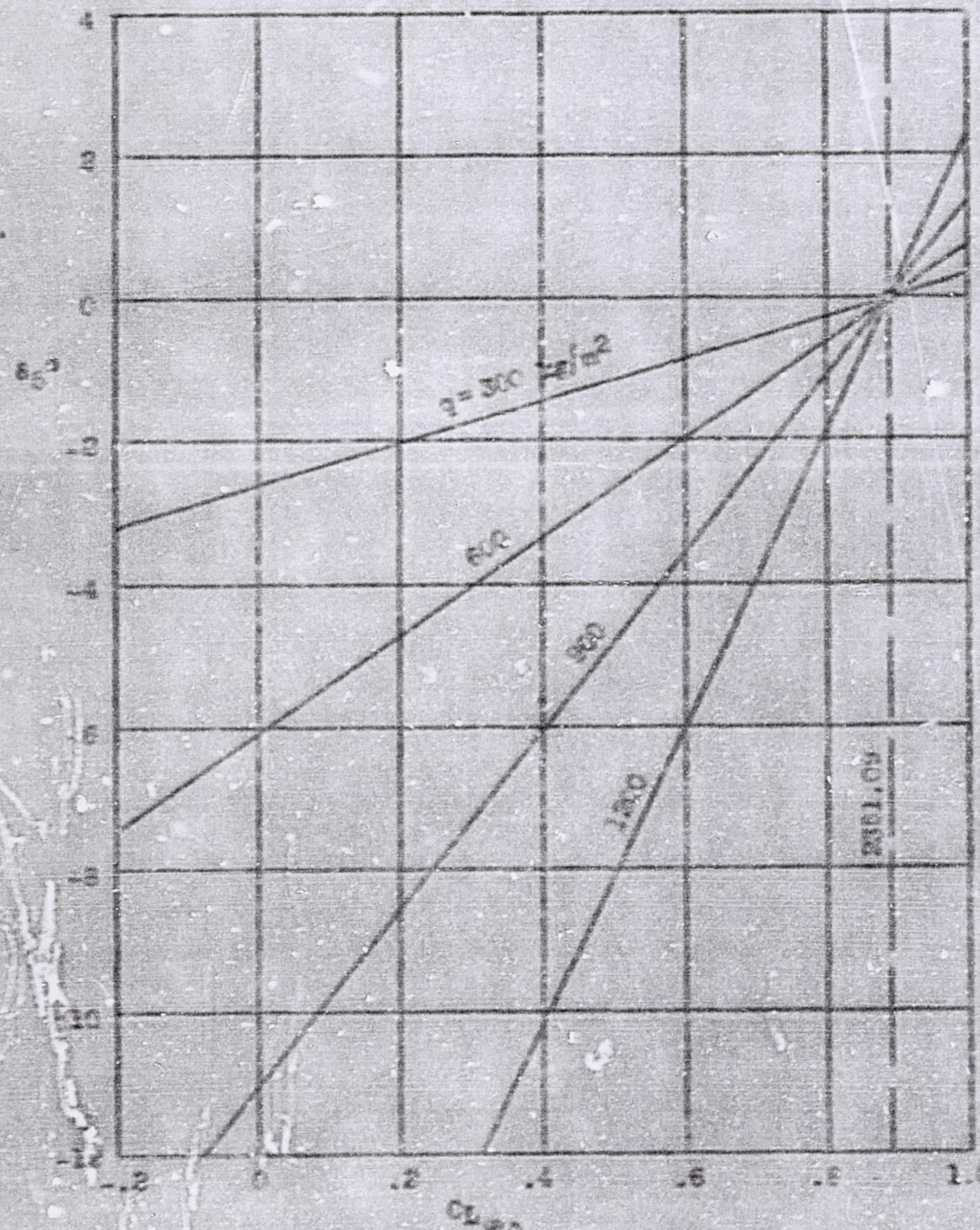
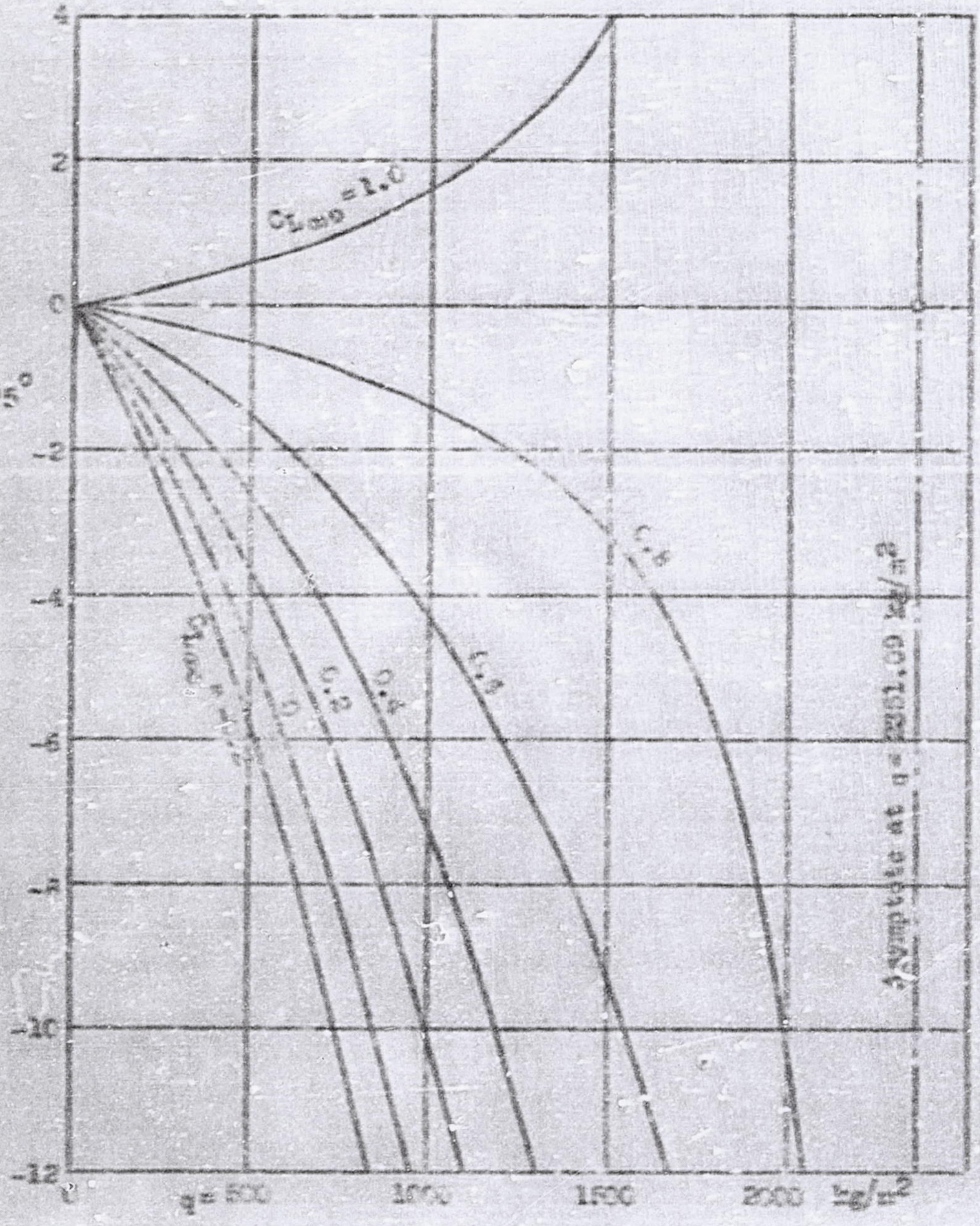


Figure 7.- Variation of δ_S with $C_{L_{m_0}}$ and q.

(Figure continued on next page)



Continuation of Fig. 7

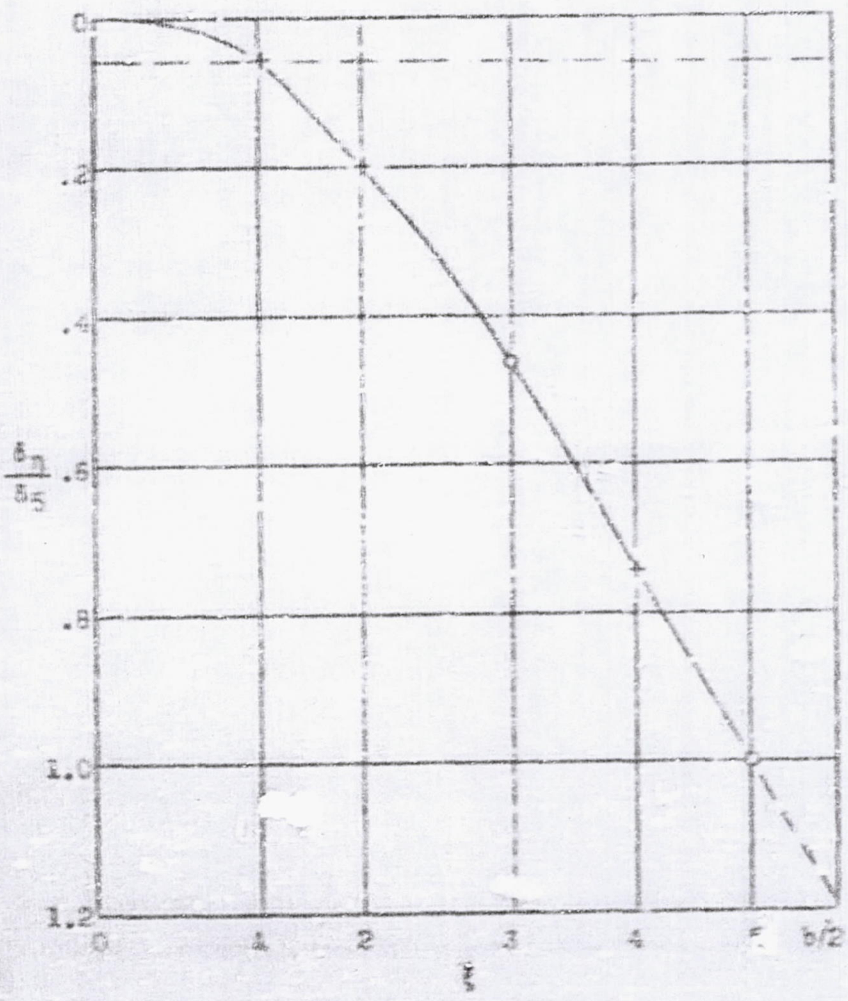


Figure 8.- Variation of δ_η/δ_5 with distance from the wing root.

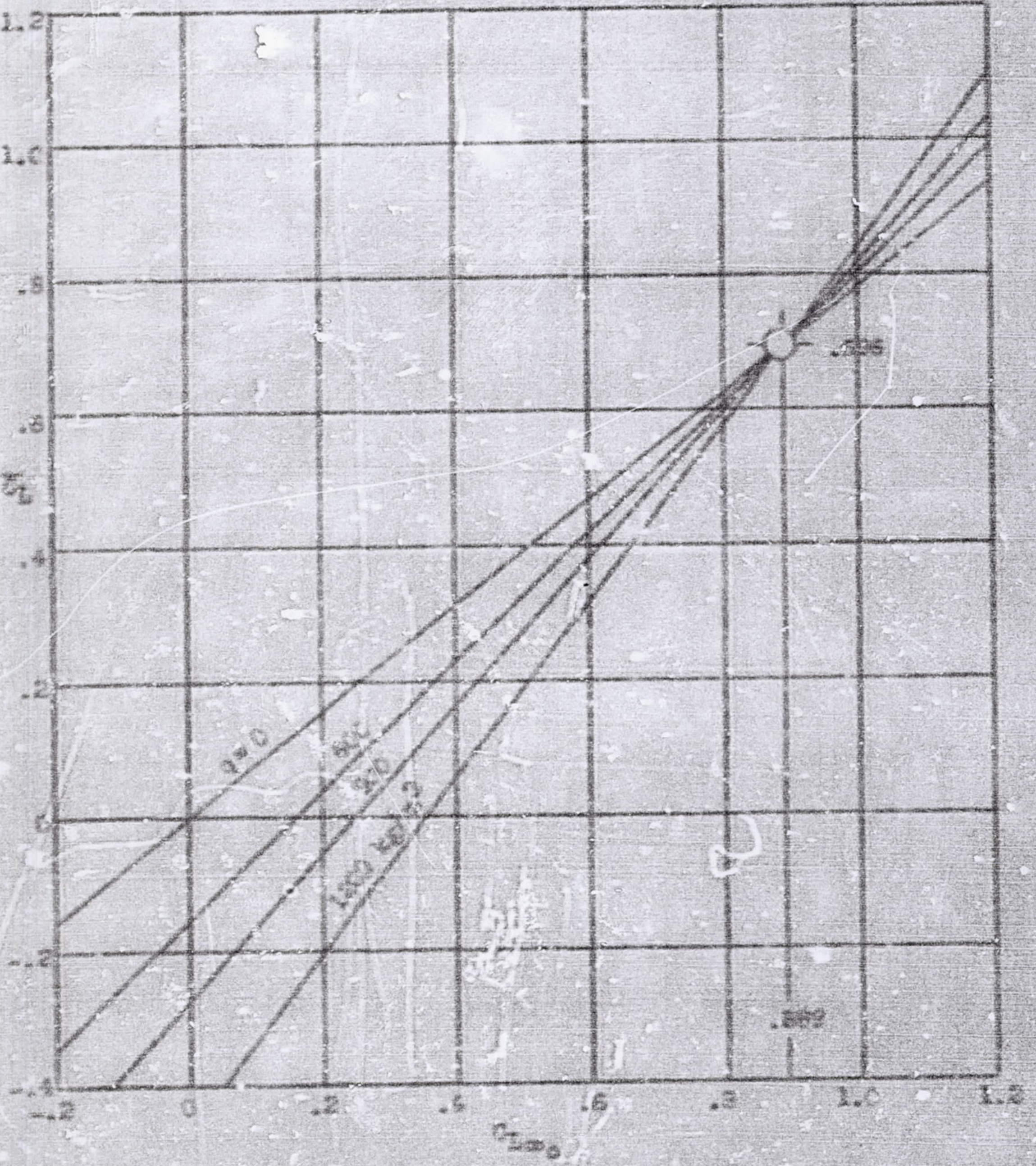


Figure 9.- Variation [illegible] lift coefficient of entire wing) with [illegible]

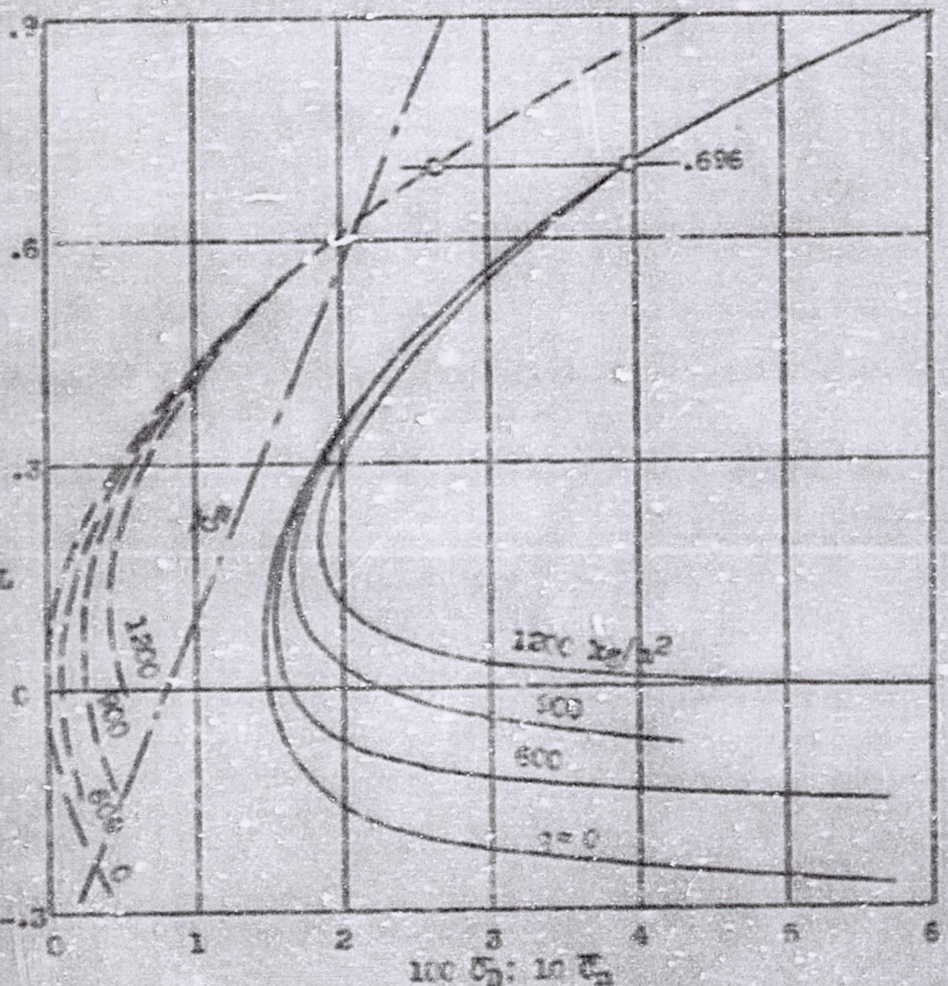


Figure 10.- Variation of $\overline{C}_L$ (absolute lift coefficient of entire wing) with $\overline{C}_D$ and $\overline{C}_m$.